## Universality of the Riemann zeta-function, Mertens' theorem and the prime number theorem

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In the proof of his universality theorem for the Riemann zeta-function $\zeta(s)$, Voronin used a theorem of Markov and the prime number theorem with a certain error term. Later, Bagchi instead used a theorem of Bernstein and the prime number theorem of the form $\pi(x) = x(\log x)^{-1} + x(\log x)^{-2} + O(x(\log x)^{-3})$. In this talk, we will prove the modern form of the universality theorem for $\zeta(s)$ by modifying Bagchi's proof and using the prime number theorem (without an error term) $\pi(x) \sim x(\log x)^{-1}$, which is equivalent to $\sum_{p\leq x} p^{-1}\log p = \log x + c_0 + o(1)$. We will also prove Voronin's original universality theorem by using Mertens' theorem $\left|\sum_{p\leq x} p^{-1}\log p - \log x\right| < 2$, which is weaker than the prime number theorem.