Almost periodic character of some error terms level sets in prime number theory

Olivier Ramaré

CNRS, Aix-Marseille Université (France)

A main player in this talk will be the function defined for positive t by $\phi(t) = (\psi(e^t) - e^t)/e^{-t/2}$ where $\psi(x)$ is the summatory function of the van Mangold function. A proper definition of $\phi(t)$ for negative t will be provided.

Our subject of interest are the sets $E(y) = \{t \in \mathbb{R}, \phi(t) > y\}$, under the generalized Riemann Hypothesis. More than 20 years ago we proved with J. Kaczorowski that such sets have some strong almost periodic character, a study completed in an unpublished note of J.C. Schlage-Puchta where it is shown that indeed these sets (or more precisely their characteristic functions) are almost periodic in the L^2-Stepanov sense (except for y in some at most countable subset of real numbers). The first object of this talk is to set this background and to recall these results.

The second object of this talk is to detail some open questions. A major one is this: what are the periods of the set $E(y)$? We showed that they lie in the $\mathbb{Q}$-vector space generated by the ordinates of the zeroes of the Riemann zeta-function, a poor man's conjecture is that these ordinates should exactly be the periods but the numerical knowledge is missing to confirm such a conjecture.

This works readily extends to the Selberg class.