

The Kummer ratio

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Let q be a prime and consider the cyclotomic field $\mathbb{Q}(\zeta_q)$. The ratio of its class number to the class number of its maximal real subfield $\mathbb{Q}(\zeta_q + \zeta_q^{-1})$ is known as the *relative class number* and is denoted by $h_1(q)$.

In 1851, Kummer claimed that $h_1(q)$ is asymptotically equal to an explicit elementary function $G(q)$. The quotient

$$R(q) := \frac{h_1(q)}{G(q)}$$

is now known as the *Kummer ratio*. Understanding the extent to which $R(q)$ deviates from 1 has proved to be a subtle problem, closely connected with deep questions concerning the distribution of prime numbers.

In this talk, I will survey recent advances on the irregular behavior of the Kummer ratio and discuss related phenomena arising in the study of Euler–Kronecker constants associated with the fields $\mathbb{Q}(\zeta_q)$ and $\mathbb{Q}(\zeta_q + \zeta_q^{-1})$.

The talk is based on joint work with Neelam Kandhil, Alessandro Languasco, Pieter Moree and Alisa Sedunova.