Zeta Functions and Their Classes: Results and Problems in Various Aspects
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Some results on universality for the Selberg–Steuding class

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Let $\mathcal{S}$ be the Selberg class, i.e. the class of Dirichlet series satisfying the Ramanujan hypothesis, having a meromorphic continuation with at most a pole at $s = 1$, a functional equation of Riemann type, and an Euler product. Then the Selberg–Steuding class $\mathscr{S}$ consists of those L-functions from the class $\mathcal{S}$ that additionally satisfy the prime mean-square condition of the form

$$\lim_{x\to\infty} \frac{1}{\pi(x)} \sum_{p\leq x} |a(p)|^2 = \kappa > 0,$$

a kind of prime number theorem for the Dirichlet coefficients.

In 1975, Voronin proved that the shifts of the Riemann zeta-function $\zeta(s + i\tau)$ approximate uniformly on compact subsets of the right half-plane of the critical strip any non-vanishing analytic function. In this talk, we discuss some universality results for the Selberg–Steuding class $\mathscr{S}$.